Electromagnetic induction: It is the production of current or voltage whenever there is a relative motion between the conductor and the magnetic field. This process causes induction of e.m.f in the circuit. The induced e.m.f depends on the speed (motion) of the magnet, number of turns on the coil and the strength of the magnet. Magnetic line of force is produced by the magnet, The Induced e.m.f (ε) = Blv
where B, v and the conductor are all perpendicular to one another.
The induced e.m.f increases with increasing of
- (i) the speed of motion of the conductor or the magnet
- (ii) the number of turns on the coil and
- (iii) the strength of the magnet.

Experiment to verify laws of electromagnetic induction
1. A solenoid is produced by winding several turns of coils of insulated copper wire. Connect the solenoid to a galvanometer. Move the n-pole of the magnet toward the end of the coil. Note the direction of the galvanometer. Move the magnet away from the coil and note the new direction. Then, keep the magnet stationary in the coil. When a bar of magnet is drived into a coil connected to a galvanometer, the galvanometer deflects in a particular direction as shown in figure A.
2. Current will flow in the galvanometer whenever there is relative motion between the coil and the magnet (when magnet move toward or away from the coil). Such current is called induced current. When the magnet remained stationary in the coil, the galvanometer gives no deflection showing that no current is flowing through it.

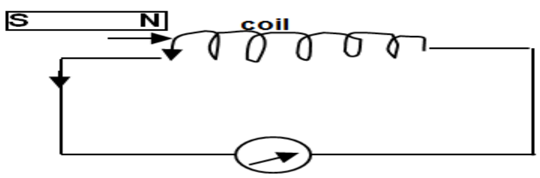


Figure A

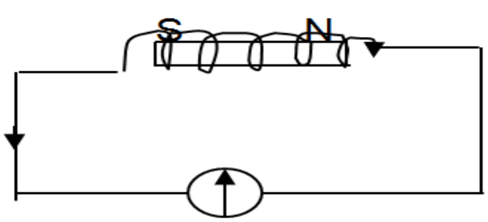


Figure B

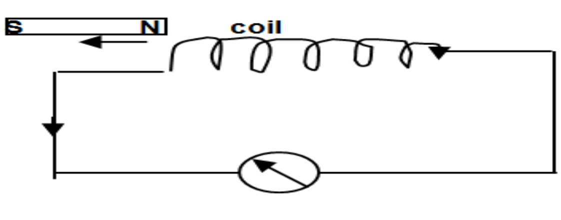


Figure C

(i) Where there is a relative motion between a closed circuit and a magnet an induced current or e.m.f is generated in the coil.

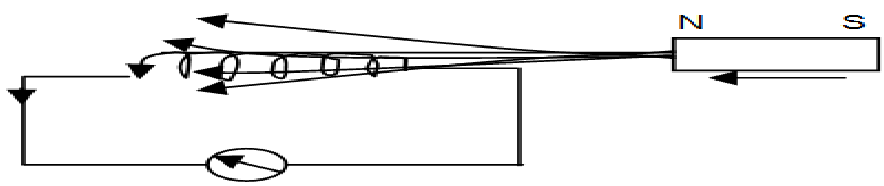
(ii) The direction of the induced current depends on the polarity and movement of the magnet.

(iii) The faster the relative motion , the greater size of the induced current.

(iv) Increase in the strength of the magnet and increase in the number of turns of the coil all increase the size of the induced current.

Faraday 's Law of Electromagnetic Induction

Faraday's Law



1st law:- An e.m.f is induced whenever there is flux leakage. When the magnetic flux threading a circuit is changing an e.m.f is induced in the circuit.

2nd law:- The magnitude of the induced e.m.f is proportional to the rate of change of the magnetic flux linking the circuit. $\varepsilon = - N \frac{(\Delta BA)}{\Delta t}$

The magnetic flux $\Phi = BA$

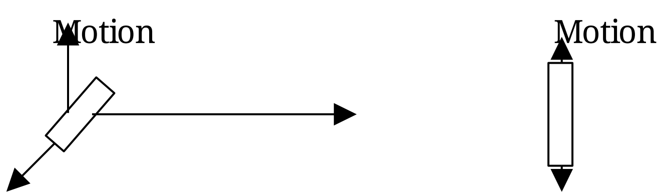
where A is the cross-sectional area of the coil and N is the number of turns of the coil

$\varepsilon = - \frac{\Delta Q}{\Delta t}$

Lenz's Law: The direction of the induced emf and current opposes the motion or change producing it. The minus sign in Faraday's law is consequence of the **Lenz's Law**

Direction of induced Current (Flemming Rule)

Right hand rule states that if the thumb, the fore finger (F) and the middle finger (I) of the right hand are held at right angles to each other and if F points in the direction of field, thumb is in direction of motion, then I points in the direction of the induced current.



Max. Current F

Conductor or current Field (Zero Current)

Example

A coil of 10 turns and cross-sectional area $5cm^2$ is at right angles to a flux density 2×10^{-2}T which is reduced to zero in 10s. Find the flux change and the induced e.m.f.

Solution

The flux linkage = NBA = $10 \times 2 \times 10^{-2} \times 5 \times 10^{-4}$

(N = number of turns in the coil, β = flux density, A = cross sectional area)

Therefore, the change in the flux is:

1.0×10^{-4} Wb

The change in time is 10s, and so the induced e.m.f is:

$$\varepsilon = \frac{NBA}{t} = \frac{1.0 \times 10^{-4}}{10} = 1.0 \times 10^{-5}\text{V}$$

Motors and Generators (Dynamo)

The moving-coil instrument has a rectangular coil, which rotates in the narrow space between a soft-iron cylinder and curved pole-pieces of a horse-shoe magnet. Springs control the deflection. The electric motor has a coil wound on an armature between the poles of a magnet and is similar in principle to a moving-coil meter. The ends of the coil are joined to the halves of a split-ring commutator. The commutator rotates with the coil and presses continuously against fixed brushes, to which a battery is connected. The generator has a coil that is driven round at constant speed between the poles of a magnet. An induced voltage is then set up at the terminals, so that the generator acts like a battery.

The principles of the motor and generator

When a coil is rotated between the poles of the permanent magnet, a current is induced in the coil. Two slip rings are connected to the ends of the coil. Two carbon brushes make contact with the slip rings. Current induced in the coil passes out through one brush during one half cycle and out through the other brush during the second half-cycle. The current produced is therefore alternating current; this is a simple a.c. dynamo or generator. By replacing the slip rings with a split ring commutator the a.c. dynamo is then converted to a d.c. dynamo.

By passing a current through a d.c. dynamo, the dynamo will become an electric motor. The constructions of both are the same. In the case of a motor, electrical current is supplied to turn the coil whereas in the dynamo the coil is turned, usually by mechanical means and electric current is produced.

The simple dynamo produces a low emf. To increase the emf and produce a practical instrument then wind the coil on a soft iron armature; increase the number of turns in the coil; turn the coil faster and increase the strength of the field magnet

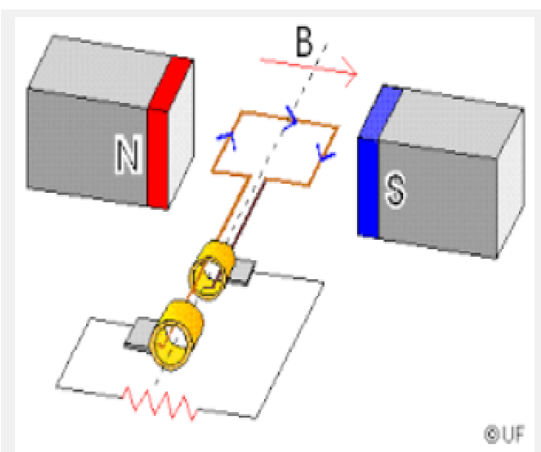
A.C. Generator

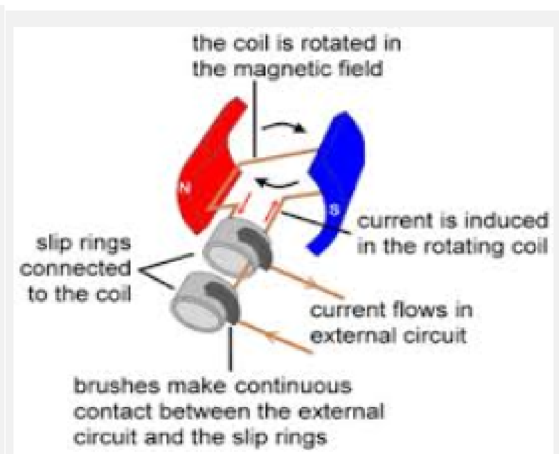
A.C. generator (alternating current) is a device for converting mechanical energy into electrical energy. The mechanical energy is produced by the motion of the coils of wire in a magnetic field. The electrical energy is derived from the e.m.f current, which is subsequently induced in the coil. It is alternating because it flows in one direction during one part of the cycle and in the opposite direction during the rest of the cycle. It operates on Fleming's right hand rule

The a. c. generator consists of;

(i) A magnet which provides a strong magnetic field
(ii) An armature consisting of several turns of wire wounds on a soft iron core.
(iii) Two slip rings on which two carbon brushes rest.

The armature is free to revolve on the axis between the poles of the magnet. The slip rings are connected to the ends of the armature. The two carbon brushes lead current away from the ring into external circuit.



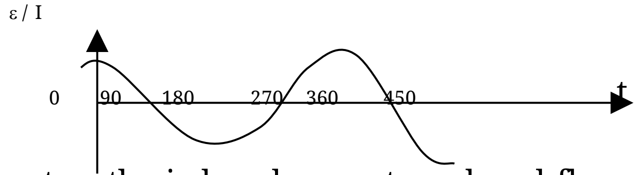


Shematic of a.c. generator

As the armature coil rotates between the magnets, the magnetic lines of force through the armature coil change continuously and current is induced in the coil.

The current on both sides of the coil is the same when in horizontal position. No lines of force

link it but rate of change of flux is maximum. Hence maximum induced e.m.f. is available at the terminals. In vertical position, lines of force are maximum but rate of change of flux is zero in between the current / e.m.f.



D.C. Generator

For the d.c. generator the induced current produced flows only in one direction. To convert a.c generator to d.c.generator the slip rings are replaced by split rings or commutator , and direct current is produced. To increase the magnitude of the direct current, many coils plus commutator with many segments are used.

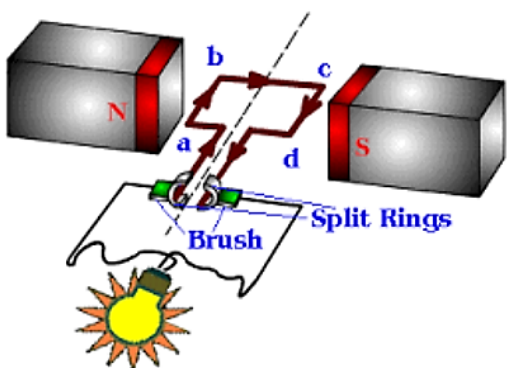
To increase a.c. or d.c. current;

(i) Increase the number of turns in the coil

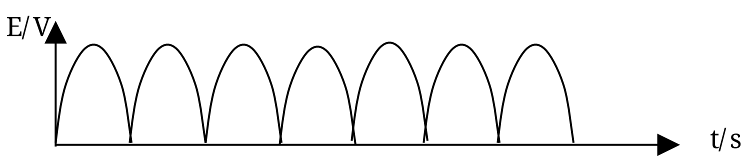
(ii) Increase the flux (use powerful magnets)

(iii) Winding the coil on soft iron core

(iv) Increasing the rate of rotation of the coil



DC generator



Current in d.c. generator

Self Inductance

When the current in a circuit changes, the magnetic field enclosed by the circuit also changes, and the resulting change in flux leads to a self-induced e.m.f given as:

Self induced emf $\varepsilon = -L \frac{dI}{dt}$

where $\frac{dI}{dt}$ is the rate of change of the current and L is the property of the circuit called its self-inductance. The minus sign indicates that the direction of ε is such as to oppose the change in the current dI that caused it. Unit of inductance is the henry (H)

The energy E stored in an inductor is given by $E = \frac{1}{2} LI^2$

Mutual Inductance

When two coils are in proximity, the mutual inductance M, between the coils is defined as the induced e.m.f in one coil when the current changes in the second coil. That is;

$$\left\{ \begin{array}{l} emf \text{ induced in coil 1 by} \\ \text{current changing in coil 2} \end{array} \right\} = m \times \left\{ \begin{array}{l} rate \text{ of change of} \\ \text{current in coil 2} \end{array} \right\}$$

Thus $\varepsilon_2 = m \frac{dI_1}{dt}$ and $\frac{dQ_2}{dt} = M \frac{dI_1}{dt}$

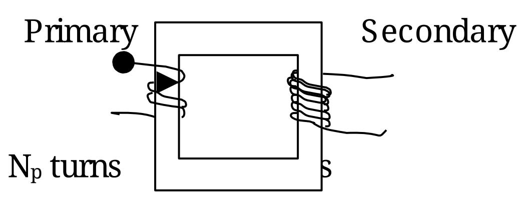
The Transformer

Transformer is a device that can be used to increase (step-up) or decrease (step-down) voltage. In the step-down transformer, the number of turns in the primary coil is more than the number of coil in the secondary coil. When it is used to step-up voltage, the number of turns in secondary coil are more than the number of turns in primary coil. A transformer changes alternating voltage from low to high values and vice versa. It consists of

(i) a primary coil to which the voltage V_p is connected
(ii) a secondary coil (output) from which the new voltage V_s is obtained.
(iii) a soft iron laminated core on which both coils are wound.

<u>Ways by which energy is lost in a transformer</u>

(1) Heat losses: this is brought about as a result of wire of high resistance. It can be minimized using wire of low resistance
(2) Flux leakage: this is brought about as a result of non-efficient core. It can be minimized by using designing efficient core.
(3) Eddy current: this is caused by magnetic flux variation. It can be minimized by laminating the iron core.
(4) Hysteresis:



$$\frac{\textbf{Secondary e.m.f}}{\textbf{primary e.m.f}} = \frac{\textbf{no of turns in the secondary coil}}{\textbf{no of turns in the primary coil}}$$

$$\frac{E_s}{E_p} = \frac{N_s}{N_p}$$

If no loss of energy the power going into a transformer $P_p = I_pV_p$ must be equal to the power $P_s = I_sV_s$ going out of the transformer

Example: A 100 turn coil whose resistance is 6Ω encloses an area of 80 cm². How rapidly should a magnetic field parallel to its axis change in order to induce a current of 1mA in the coil?

Solution

The required emf $\varepsilon = IR = 10^{-3} \times 6 = 6 \times 10^{-3}$V

Area of a turn is $A = \frac{80 \text{ cm}^2}{(100 \text{ cm/m})^2} = 8 \times 10^{-3}$ m²

$$\varepsilon = N\frac{\Delta(BA)}{\Delta t} = NA\frac{\Delta B}{\Delta t}$$

$$\frac{\Delta B}{\Delta t} = \frac{\varepsilon}{NA} = \frac{6 \times 10^{-3}}{100 \times 8 \times 10^{-3}} = 0.0075 \text{ Ts}^{-1}$$

Example

A transformer connected to a 120V ac power line has 200 turns in its primary winding and 50 turns in its secondary winding. The secondary is connected to a 100Ω light bulb. How much current is drawn from the 120V power line?

Solution

The voltage across the secondary is given as:

$$V_s = \frac{N_s}{N_p}V_p = \frac{50}{200} \times 120 = 30\text{V}$$

The current $I_s = \frac{V_s}{R} = \frac{30}{100} = 0.3$A

Also current in the primary is given as

$$I_p = \frac{N_s}{N_p}I_s = \frac{50}{200} \times 0.3 = 0.075\text{A}$$

Example

(i) How much energy is stored in a 20-mH coil when it carries a current of 0.2A?

(ii) What should the current in the coil be in order that it contains 1J of energy?

Solution

(i) Energy $W = \frac{1}{2}LI^2 = \frac{1}{2} \times 20 \times 10^{-3} \times (0.2)^2$ J

$W = 4 \times 10^{-4}$J

(ii) $= \sqrt{\frac{2W}{L}} = \sqrt{\frac{2 \times 1}{20 \times 10^{-3}}} = 10$A

Example

A 0.1H inductor whose resistance is 20Ω is connected to a 12V battery of negligible internal resistance (i) what is the initial rate at which the current increases (ii) what happens to the rate of current increases (iii) what is the final current?

Solution

(i) Induced emf ε is in the opposite direction to the battery E. Thus the p.d across the inductor at any time is

$$IR = E - \varepsilon = E - L\frac{dI}{dt}$$

Initially t = 0, I = 0 thus $E - L\frac{dI}{dt} = 0$ and $\frac{dI}{dt} = \frac{E}{L} = \frac{12}{0.1} = 120\ As^{-1}$

(ii) Since $\frac{dI}{dt} = \frac{E - IR}{L}$; thus as the current I increases its rate of change $\frac{dI}{dt}$ decreases.

(iii) When the current has reached its final value $\frac{dI}{dt} = 0$ and $I = \frac{E}{R} = \frac{12\,V}{20\,\Omega} = 0.6\,A$

Example

(a) Calculate the inductance of a solenoid containing 250 turns if the length of the solenoid is 20.0cm and its cross-sectional area is $4.00\times10^{-4}m^2$.

(b) Calculate the self-induced e.m.f. in the solenoid described in part A if the current through it is decreasing at the rate of 40.0A/s.

Solution:

(a) The inductance can be found from

$$L = \frac{N\phi_B}{I}$$

The flux through each turn is

$$\phi_B = BA,\ \text{and}\ B = \mu_o nI = \mu_o \frac{N}{\ell} I$$

$$= 157{,}079.63\times10^{-9}\ T.m^2/A = 0.157\times10^{-3}\ T.m^2/A = 0.157\ mH$$

(b) $\frac{\Delta I}{\Delta t} = -40.0\ A/s$

$$\varepsilon = -L\frac{\Delta I}{\Delta t} = -(1.57\times10^{-4}\ H)(-40.0\ A/s) = 6.28\ mV$$

Example

An AC generator consist of 10 turns of wire of area A $=0.08m^2$ with a total resistance of 16.0Ω. The loop rotates in a magnetic field of 0.600T at a constant frequency of 50.0Hz.

(a) Find the maximum induced e.m.f.

(b) What is the maximum induced current?

(c) Determine the induced e.m.f. as a function of time.

(d) Determine the time variation of the induced current.

Solution:

(a) $\omega=2\pi f=2\pi(50.0Hz)=100\pi=314rad/s$

$E_{max} = NAB\omega = 10(0.08m)(0.6T)(314rad/s) = 151V$

(b) From Ohm's law and the result in part A, we find that

$$I_{max} = \frac{E_{max}}{R} = \frac{151V}{16.0\ \Omega} = 9.4A$$

(c) $E = E_{max}\sin\omega t = (151V)\sin 314t$ where t is in seconds

(d) $I = I_{max}\sin\omega t$

$=(9.4A)\sin 314t$

Example

A long, straight wire carries a current of 4.00A. At one instant, a proton, 5.00mm from the wire travels at 2.00×10^3 m/s parallel to the same direction as the current. Find the magnetic force that is acting on the proton because of the magnetic field produced by the wire.

Solution:

$$B = \frac{\mu_o I}{2\pi r} = \frac{(4\pi\times10^{-7}\,T.\,m/A)(4.00A)}{2\pi(5.00\times10^{-3}\,m)} = 1.60\times10^{-4}\,T$$

This field is directed into the page at the location of the proton (right-hand rule).

The magnitude of the magnetic force on the proton is:

$F = qvB = (1.60x10^{-19}C)(2.00x10^3 m/s)(1.60x10^{-4}T)$

$=5.12x10^{-20}N$

The force is directed at right angle toward the wire

Example

(11) A transformer on a utility pole operates at V_p=5.8kV on the primary side and supplies energy to a lot of houses that are nearby at V_s=102V, both quantities being r.m.s. values. Assume an ideal step-down transformer, a purely resistive load, and a power factor of utility.

(i) Find the turns ratio N_P/N_S of the transformer.

(ii) Find the r.m.s. currents in the primary and secondary of the transformer if the average rate of energy consumption (or dissipation) in the houses served by the transformer is 87kW.

(iii) Find the resistive load R_s in the secondary circuit and the corresponding resistive load R_p in the primary circuit.

Solution:

(i) $\frac{V_S}{V_P} = \frac{N_S}{N_P}$

Inverting both sides we have

$$\frac{N_P}{N_S} = \frac{V_P}{V_S} = \frac{5.8\times10^3\,V}{102\,V} = 56.86 \approx 57$$

(ii) The power factor cosΦ is unity for a purely resistive load; thus, with an r.m.s. current I_{rms} in the load, energy is supplied and dissipated at the average rate of

$P_{avg} = \xi I = IV$

In the primary circuit, with V_p = 5.8kV,we have

$$I_p = \frac{P_{avg}}{V_p} = \frac{87 \times 10^3 W}{5.8 \times 10^3 V} = 15A$$

Similarly, in the secondary circuit

$$I_S = \frac{P_{avg}}{V_S} = \frac{87 \times 10^3 W}{102 V} = 852.941 A \approx 852.9A$$

(iii) We can relate the resistivity of the load to the r.m.s. voltage and current with V=IR, for both circuits.
For the secondary circuit, we have

$$R_s = \frac{V_s}{I_s} = \frac{102V}{852.941A} = 0.1196 \ \Omega \approx 0.12 \ \Omega$$

Similarly, for the primary circuit, we have

$$R_p = \frac{V_p}{I_p} = \frac{5.8 \times 10^3 V}{15A} = 387 \Omega \approx 390 \Omega$$

(i) **Faraday's law** states that an EMF is induced in a coil when there is a change in the magnetic flux through the coil. EMF = $-N\Delta\Phi/\Delta t$
(ii)**Lenz's law** state that the induced current produced by induced EMF in a coil flows in a direction so as to oppose the charge producing it.
(iii) **Mutual inductance** occur when EMF is induced in coils due to change in current flowing in the coils
EMF induced = $M\Delta I/\Delta t$, M = constant of proportionality of mutual inductance measure in Henry, ΔI= change in current
(iv) **Self inductance** occur when a changing current in a coil leads to EMF induction in a cell, the induced EMF = $L\Delta I/\Delta t$ where L is the constant of proportionality.

Biot-Savart law states that the magnetic field dB at point P due to current carrying element ds carrying a steady current I is given as.

$$dB = \frac{\mu_0 I dl \sin\theta}{4\pi x^2}$$

Where x is the distance from the element to the point P, μ_0 = permeability of free space.

The expression for magnetic field in a straight wire conductor $B = \mu_0 I/2\pi x$
The expression for magnetic field in an ideal solenoid, $B=\mu_0 NI/L$,
where r is the radius of the coil
N= no of turn of coil,
L = length of the coil